Control of Fluid Fronts in Potential Flow
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- Control of displacement fronts and/or fluid parcels using variable rate, from multiply perforated horizontal wells.
- Determine injection strategy to steer the front according to pre-determined dynamics.
- Adaptive control, as history matching improves reservoir knowledge.
- Potential flow, unit-mobility ratio displacement greatly facilitates calculations.

Problem Statement

Given a front description \( F(x, y, t) = x - f(y, t) = 0 \), find the rate profile at the injection well, to accomplish the desired front dynamics (front steering).

- Potential flow and unit-mobility ratio displacement allows for pressure field to be computed using Green’s function (e.g. by the method of images, separation of variables).

- Kinematic equation for the front satisfies

\[
p(x, y, t) = \int \frac{G(0, \eta; x, y) \alpha(\eta, t)}{\eta} d\eta
\]

- Substitution of the pressure field leads to a Fredholm equation for the solution of the injection rate profile, as a function of time

\[
f_f(y) = \int \frac{K(\eta, f(y, t), y) \alpha(\eta, t)}{\eta} d\eta
\]

Where the kernel is

\[
K(\eta, f(y, t), y) = f_f(y, t) \frac{\partial G}{\partial \eta}(0, \eta; f(y, t), y) - \frac{\partial G}{\partial x}(0, \eta; f(y, t), y)
\]

The Green Function is defined as

\[
G(0, \eta; x, y) = \frac{2 \sinh(\eta, (1 - x) \cos(\xi, y)) \cos(\xi, \eta)}{\xi, \cosh(\xi, \eta)}
\]

where \( \xi = n \pi \)

- Solution of the Fredholm equation allows for the control profile to be determined, hence for the front to be steered according to prescribed dynamics.

- An example for Homogeneous case:

- Approach can be extended to heterogeneous reservoirs, where now the Green’s function needs to be determined numerically, i.e.

Extensions:

\[
\nabla \cdot (k(x, y) \nabla G) = \delta(x - \xi, y - \eta)
\]

\[
\frac{\partial G}{\partial \xi} \bigg|_{\xi, \eta} = 0 \ldots \frac{\partial G}{\partial \eta} \bigg|_{\xi; \eta, \xi} = 0 \ldots G(\xi = 1, \eta) = 0
\]

- Substitution of the pressure field leads again to a Voltera equation for the solution of the injection rate profile, now in terms of the field heterogeneity

\[
f_f(y) = \int \frac{K(\eta; f(y, t), y; k(f, y)) \alpha(\eta, t)}{\eta} d\eta
\]

- Solution of the Fredholm equation allows for the control profile to be determined, hence for the front to be steered according to prescribed dynamics.

Useful cases are being studied

- Stabilize a protrusion of the front through rate control
- Steer front to by-pass a certain region or channel