Unified Analysis of Diagnosis Methods for Process Monitoring
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Introduction
Statistical process monitoring (SPM) is used in industry to detect and diagnose abnormal behavior of processes. A popular method used in SPM is principal component analysis (PCA). A fault is detected when one of the fault detection indices is beyond its control limit. A fault diagnosis method is used to identify the cause of the fault. Several methods, that involve the contribution of a variable to the fault detection index, have been proposed.

Unified Diagnosis Methods

Fault diagnosis methods
Decompose the index as summation of variable contributions. They are widely used in industry:

\[ CDIC_{\text{index}} = x^T M_{\text{c}} \xi_i \]
\[ PC_{\text{index}} = x^T M_{\text{c}} \xi_i \]

Partial Decomposition Contributions
Partially decompose the index as the summation of variable contributions:

\[ DCIC_{\text{index}} = x^T M_{\text{c}} \xi_i \]

Reconstruction-Based Contributions
Use the amount of reconstruction of an index along a variable direction:

\[ RBC_{\text{index}} = x^T (M_{\text{c}} \xi_i) \]

Diagonal Contributions
Measure the amount of reconstruction of an index along a variable direction:

\[ DCIC_{\text{index}} = x^T (M_{\text{c}} \xi_i) \]

General Decompositive Contributions

\[ GDC_{\text{index}} = x^T M_{\text{c}} \xi_i \]

Diagonal Contributions

\[ DC_{\text{index}} = x^T (M_{\text{c}} \xi_i) \]

Analysis of Diagnosability

Simple fault
Fault in sensor j with large magnitude: \( x = \xi_j \)

Notation:
\( \xi_j M_{\text{c}} = [M_{\text{c}}]_{ij} \)
\( \xi_j = 0 \)
\( \xi_j M_{\text{c}} = [M_{\text{c}}]_{ij} \)

General Decompositive Contributions

\[ GDC_{\text{index}} = \left( [M_{\text{c}}]_{ij} \right)_{i \neq j} \text{ for } i \neq j \]
\[ \left( [M_{\text{c}}]_{ij} \right)_{i = j} \text{ for } i = j \]

Correct diagnosis if
\[ [M_{\text{c}}]_{ij} \leq [M_{\text{c}}]_{ij} \]

Correct diagnosis NOT guaranteed!

\[ [M_{\text{c}}]_{ij} > [M_{\text{c}}]_{ij} \]

Correct diagnosis guaranteed!

Reconstruction-based Contributions

\[ RBC_{\text{index}} = \left( \xi_j M_{\text{c}} \right)_{j \neq i} \text{ for } i \neq j \]
\[ \left( \xi_j M_{\text{c}} \right)_{i = j} \text{ for } i = j \]

Correct diagnosis if
\[ \xi_j M_{\text{c}} \leq [M_{\text{c}}]_{ij} \]

Correct diagnosis guaranteed!

Simulation

The objective is to compare the rate of correct diagnosis for medium size faults given by the diagnosis methods.

Model

\[ x_1 = \begin{bmatrix} -6.3079 \ 0.7218 \ -0.2297 \\ -4.9757 \ 0.3216 \ 0.2247 \end{bmatrix} \]
\[ x_2 = \begin{bmatrix} -6.3032 \ 0.7261 \ -0.3496 \\ -6.4411 \ -0.0212 \ 0.0601 \end{bmatrix} \]
\[ x_3 = \begin{bmatrix} -6.3318 \ 0.7272 \ -0.1215 \\ -8.3266 \ 0.4616 \ -0.1545 \end{bmatrix} \]

Data:

- \( x_1, x_2, x_3 \) are random variables with std of 0.8 and 0.6
- The noise is Gaussian zero-mean with std of 0.2
- 1000 samples used to build the model
- Faults are of the form: \( \xi_j M_{\text{c}} = [M_{\text{c}}]_{ij} \)
- \( y \) is uniformly random between 0 and 5
- \( \xi_j \) is uniformly random out of the six possible variable directions

Results:

- 2000 faults are simulated
- 3 PC are used to build the PCA model

Conclusions and Acknowledgements

The different diagnosis methods presented here can be classified in three general types: Decompositive contributions, Reconstruction-based contributions, and Diagonal contributions. The diagnosis method mostly used in industry: Complete Decomposition contributions, DO NOT guarantee correct diagnosis for simple faults, while other methods like RBC, Diagonal and Partial Decomposition contributions DO guarantee correct diagnosis for simple faults. For medium size sensor faults, different diagnosis methods perform better for different indices; but, in general, RBC, using the \( \phi \) index, has the largest rates of correct diagnosis.

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Bibliography

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